# Predicting the repeated load behaviour of pavement subgrades using the three-surface kinematic hardening model

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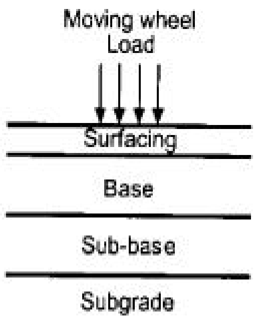
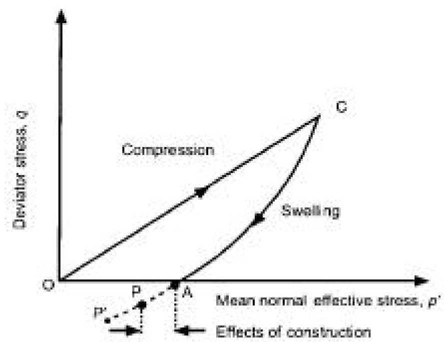
## Introduction

Pavement soil mechanics and traditional soil mechanics have developed as two separate disciplines. As a result, current pavement design methods are based largely on empirical results, whereas constitutive modelling in traditional soil mechanics is now very sophisticated and based on fundamental elasticity and plasticity theory. As yet, it is not known of any single model in pavement engineering which can predict both the resilient (quasi-elastic) response over one application of wheel load, and accumulation of permanent deformation over many cycles. The aim of this project is to apply the principles of soil mechanics, and in particular the kinematic hardening concept, to modelling the behaviour of low-volume roads under repeated loading. This paper describes some preliminary results using CRISP with the 3-SKH model to analyse a three-layer pavement. The model is capable of modelling residual and permanent deformation, but it will be shown that the predictions of shear strain are poor under low applied stress pulses: this is demonstrated using a simple triaxial simulation.

## Finite element analysis of a 3-layer pavement

A typical 3-layer pavement (Figure 1(a)) was analysed using CRISP. The pavement consisted of 100 mm of bituminous material (assumed linear-elastic) over 150 mm granular sub-base (assumed linear-elastic) over clay subgrade (3-SKH). The pavement was assumed to be constructed in a 'cut' condition with typical stress history as shown in Figure 1b.




(a) (b)

Figure 1. (a) Cross section of a pavement and (b) typical stress history of a pavement.

A mesh of 323 8-noded quadrilateral elements with 3×3 integration points were used. For the vertical boundary, radial displacements were restrained and vertical displacements were allowed. For the horizontal boundary, radial and vertical displacements were prevented. A typical wheel load of 600 kPa and radius of 0.15 m was applied under drained conditions to the surface of the bituminous layer for 50 cycles. Figure 2 shows the deformation at the surface before the 50$^{th}$ load cycle was applied, and when the load was applied and then removed. It can be seen that the behaviour over one cycle is largely resilient, but that there has been an accumulation of permanent deformation over 50 cycles.

This analysis presents a fundamental modelling problem: if the material above the subgrade is assumed to be elastic, and the nodes are common at the subgrade interface, the elastic layer will pull the soil into tension each time the pavement is unloaded. This can be overcome by calculating the stress distribution applied at the top of the subgrade during the first application of load, and then applying cycles of this stress distribution at the top of the clay alone for many cycles and calculating the permanent deformation. It must then be assumed that the granular and asphalt layers will follow the surface of the clay in the long term. Since it is the long term behaviour that is of interest, the clay has been assumed to be fully drained.

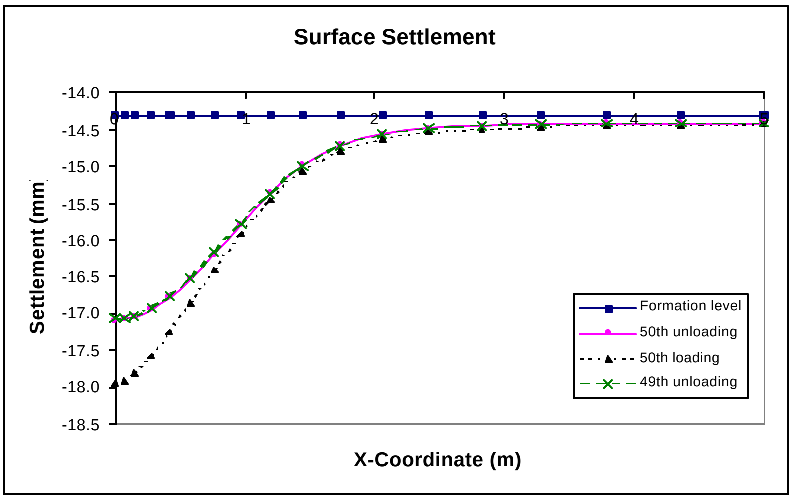



Figure 2. Permanent settlement of a pavement after 50 cycles.

Conventional pavement deformation analyses are based on layered elasticity theory, and an attempt was made to compare the results of CRISP with such an approach, which required calculating typical stress changes in the pavement using CRISP, and applying many cycles of these stress changes to triaxial samples of 3-SKH clay in order to obtain an equivalent stiffness for input to the elastic model. It was during this process that it was discovered that the 3-SKH model gives poor predictions of shear strain under low cycles of stress.

**Model predictions under cyclic loading**

Cyclic loading simulations were performed on soil isotropically normally compressed to $p'_o$ = 500kPa and the model parameters used are shown in Table 1.

| $M$ | $l^*$ | $e_{cs}$ | $k^*$ | $G_{ec}'$ (kPa) | $T$ | $S$ | $y$ |
|---|---|---|---|---|---|---|---|
| 0.89 | 0.073 | 1.994 | 0.005 | 60000 | 0.25 | 0.08 | 2.5 |

Table 1. Model parameters

Figure 3 shows the prediction of mean normal effective stress against volumetric strain for 20 cycles of isotropic load with $\Delta p'$ = 100 kPa. The result shows that there was accumulation of volumetric strain, and the rate of the accumulation of volumetric strain decreased as the number of cycles increased. The deformation caused by the first few cycles contributed most to the total deformation. After approximately 20 cycles, the rate of accumulation of volumetric strain reduced and at the 20$^{th}$ cycle, the model predicted a closed hysteresis loop.

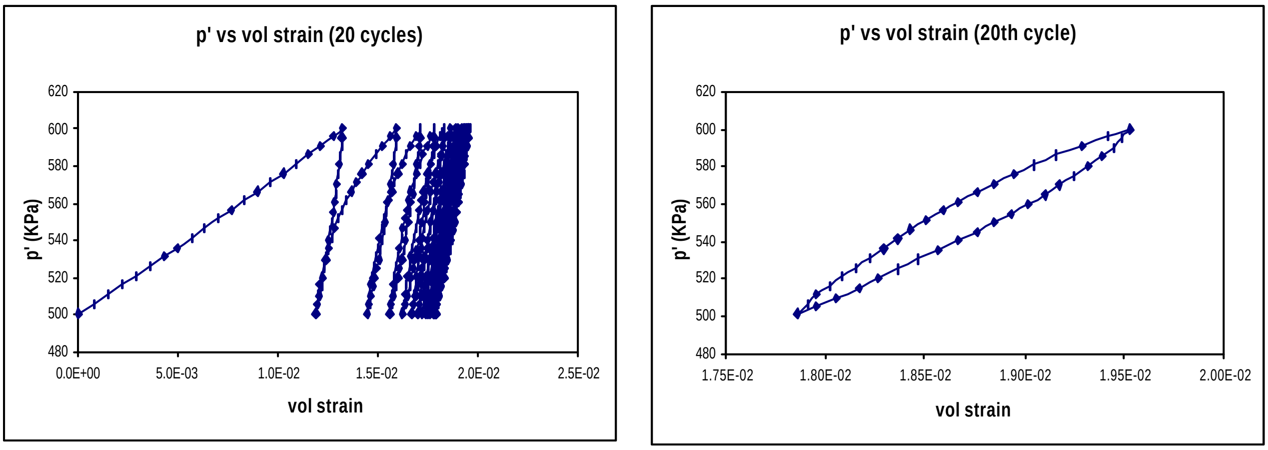
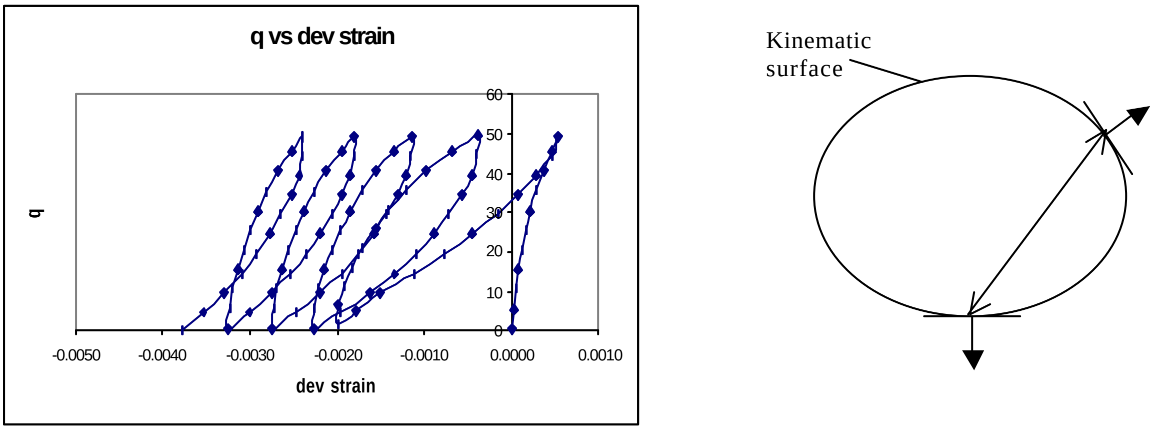



Figure 3. Mean normal effective stress against volumetric strain.

Modelling problems arose when conventional drained cyclic simulations were performed on isotropically normally consolidated soil. It was found that the model predicted the accumulation of negative permanent shear strain with each consecutive cycle if low stress pulses (relating to soil elements deep within the subgrade) were applied – see Figure 4a. The values of T and S were found to influence the results, and large cycles of stress predicted the expected accumulation of positive permanent shear strain. For small stress pulses, the reason for the accumulation of negative shear strain can be seen in Figure 4b, which shows the strain increment vectors on the yield surface on loading and unloading: the negative plastic shear strain developed during unloading was larger than the positive plastic shear strain that developed during loading.




(a) (b)

Figure 4. (a) Increasing negative permanent shear strain and
(b) position of load cycles relative to the kinematic yield surface.

This is confirmed by the $\varepsilon_p$: $\varepsilon_q$ plot shown in Figure 5. It can be seen from the graph that the magnitude of the shear strain generated during loading was less than that during unloading, and the gradient tended to zero for unloading (i.e. $d\varepsilon_p/d\varepsilon_q = 0$). Thus during unloading, a large amount of negative plastic shear strain developed.

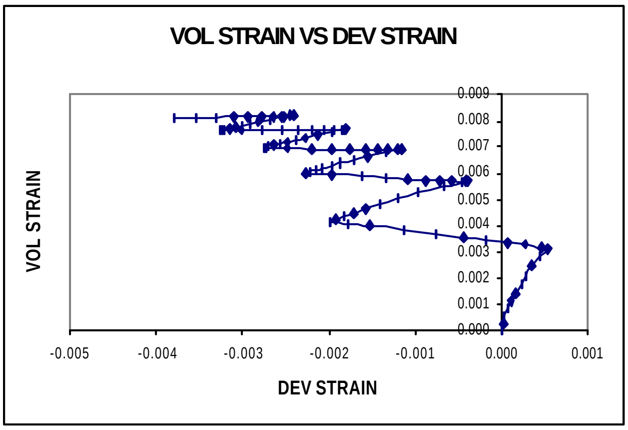



Figure 5. Volumetric strain against deviatoric strain

## Conclusion

The 3-SKH model can predict the essential features of soil behaviour under cyclic loading, and is suitable for modelling the deformation of subgrades in pavements. However, the model gives poor prediction of shear strains if the applied stress pulse is small, and this is affected by the choice of soil parameters. This should not, however, be a serious problem, because most of the deformation in a pavement will be due to strains caused by the larger stresses in the upper part of the subgrade.